Math 131 B, Lecture 1
Real Analysis

Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

Let (M,d) be a metric space. We define a new metric d' on M by

$$d'(x,y) = \begin{cases} d(x,y) & \text{if } d(x,y) < 1 \\ 1 & \text{if } d(x,y) \geq 1 \end{cases}$$

(a) [5pts.] Prove that d' is a valid metric on M.

(b) [5pts.] Show that if $S \subset M$, S is open in (M,d) if and only if S is open in (M,d').

Problem 2.

(a) [5pts.] What does it mean for a sequence $\{x_n\}$ to converge in (M, d)?

(b) [5pts.] Let $S \subset M$ and $p \in \overline{S}$. Prove that there is a sequence of points in S converging to p.

Problem 3.

(a) [5pts.] Give a definition of a compact set.

(b) [5pts.] Let S and T be subsets of a metric space M such that S is compact and T is closed in M. Prove that $S \cap T$ is compact.

Problem 4.

(a) [5pts.] Give a definition of a complete metric space.

(b) [5pts.] Which of the following metric spaces are complete? Briefly justify your answers.

- $S_1 = (0, 1] \times [2, 4)$ with the metric inherited from $\mathbb{R}^2$.
- S_2 a discrete metric space.
- $S_3 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ with the metric inherited from $\mathbb{R}^3$.

Problem 5.

(a) [5pts.] State the Lindelöf Covering Theorem.

(b) [5pts.] Let S be a set in $\mathbb{R}^n$ with the property that for every $\mathbf{x}$ in S, there is a ball $B(\mathbf{x}; r_{\mathbf{x}})$ such that $B(\mathbf{x}; r_{\mathbf{x}}) \cap S$ is countable. Prove that S is countable.